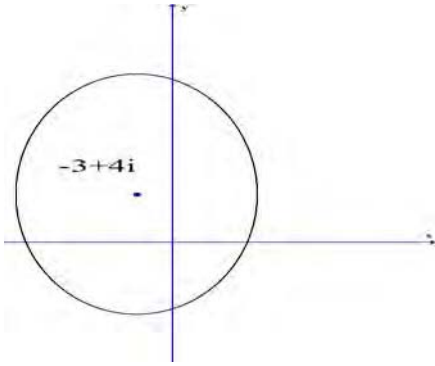


Further Pure Mathematics 2 Mark Scheme

Question	Scheme	Marks	AOs
1(a)	Consider $\det \begin{pmatrix} 3-\lambda & 1 \\ 6 & 4-\lambda \end{pmatrix} = (3-\lambda)(4-\lambda) - 6$	M1	1.1b
	So $\lambda^2 - 7\lambda + 6 = 0$ is characteristic equation	A1	1.1b
		(2)	
	So $\mathbf{A}^2 = 7\mathbf{A} - 6\mathbf{I}$	B1ft	1.1b
(b)	Multiplies both sides of their equation by $\mathbf{A}$ so $\mathbf{A}^3 = 7\mathbf{A}^2 - 6\mathbf{A}$	M1	3.1a
	Uses $\mathbf{A}^3 = 7(7\mathbf{A} - 6\mathbf{I}) - 6\mathbf{A}$ So $\mathbf{A}^3 = 43\mathbf{A} - 42\mathbf{I}^*$	A1*cso	1.1b
		(3)	
(5 marks)			
Notes:			
(a)			
M1: Complete method to find characteristic equation			
A1: Obtains a correct three term quadratic equation – may use variable other than λ			
(b)			
B1ft: Uses Cayley Hamilton Theorem to produce equation replacing λ with $\mathbf{A}$ and constant term with constant multiple of identity matrix, $\mathbf{I}$			
M1: Multiplies equation by $\mathbf{A}$			
A1*: Replaces $\mathbf{A}^2$ by linear expression in $\mathbf{A}$ and achieves printed answer with no errors			

Question	Scheme	Marks	AOs
2(i)	Adding digits $8 + 1 + 8 + 4 = 21$ which is divisible by 3 (or continues to add digits giving $2+1=3$ which is divisible by 3) so concludes that 8184 is divisible by 3	M1	1.1b
	8184 is even, so is divisible by 2 and as divisible by both 3 and 2, so it is divisible by 6	A1	1.1b
		(2)	
(ii)	Starts Euclidean algorithm $31=27 \times 1 + 4$ and $27 = 4 \times 6 + 3$	M1	1.2
	$4 = 3 \times 1 + 1$ (so hcf = 1)	A1	1.1b
	So $1 = 4 - 3 \times 1 = 4 - (27 - 4 \times 6) \times 1 = 4 \times 7 - 27 \times 1$	M1	1.1b
	$(31 - 27 \times 1) \times 7 - 27 \times 1 = 31 \times 7 - 27 \times 8$ $a = -8$ and $b = 7$	A1cso	1.1b
		(4)	
(6 marks)			
Notes:			
(i) M1: Explains divisibility by 3 rule in context of this number by adding digits A1: Explains divisibility by 2, giving last digit even as reason and makes conclusion that number is divisible by 6			
(ii) M1: Uses Euclidean algorithm showing two stages A1: Completes the algorithm. Does not need to state that hcf = 1 M1: Starts reversal process, doing two stages and simplifying A1cso: Correct completion, giving clear answer following complete solution			

Question	Scheme	Marks	AOs
3(a)	$(x - 9)^2 + (y + 12)^2 = 4[x^2 + y^2]$	M1	2.1
	$3x^2 + 3y^2 + 18x - 24y - 225 = 0$ which is the equation of a circle	A1*	2.2a
	As $x^2 + y^2 + 6x - 8y - 75 = 0$ so $(x + 3)^2 + (y - 4)^2 = 10^2$	M1	1.1b
	Giving centre at $(-3, 4)$ and radius = 10	A1ft	1.1b
		(4)	
(b)		M1	1.1b
		A1	1.1b
		(2)	
(c)	Values range from their $-3 - 10$ to $-3 + 10$	M1	3.1a
	So $-13 \leq \text{Re}(w) \leq 7$	A1ft	1.1b
		(2)	
(8 marks)			
Notes:			
(a) M1: Obtains an equation in terms of x and y using the given information A1: Expands and simplifies the algebra, collecting terms and obtains a circle equation correctly, deducing that this is a circle M1: Completes the square for their equation to find centre and radius A1ft: Both correct			
(b) M1: Draws a circle with centre and radius as given from their equation A1: Correct circle drawn, as above, with centre at $-3 + 4i$ and passing through all four quadrants			
(c) M1: Attempts to find where a line parallel to the real axis, passing through the centre of the circle, meets the circle so using “their $-3 - 10$” to “their $-3 + 10$” A1ft: Correctly obtains the correct answer for their centre and radius			

Question	Scheme	Marks	AOs																																																	
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(ii)	Identity is zero and there is closure as shown above	M1	2.1																																																	
	3 and 5 are inverses, 4 and 6 are inverses, 2 is self-inverse, 0 is identity so is self-inverse	M1	2.5																																																	
	Associative law may be assumed so S forms a group	A1	1.1b																																																	
		(6)																																																		
(b)	$4*4*4 = 4* (4 * 4) = 4 * 6$ or $4*4*4 = (4* 4) * 4 = 6* 4$	M1	2.1																																																	
	$= 0$ (the identity) so 4 has order 3	A1	2.2a																																																	
		(2)																																																		
(c)	3 and 5 each have order 6 so either generates the group	M1	3.1a																																																	
	Either $3^1 = 3, 3^2 = 4, 3^3 = 2, 3^4 = 6, 3^5 = 5, 3^6 = 0$ Or $5^1 = 5, 5^2 = 6, 5^3 = 2, 5^4 = 4, 5^5 = 3, 5^6 = 0$	A1, A1	1.1b 1.1b																																																	
		(3)																																																		
(11 marks)																																																				

Question 4 notes:	
(a)(i)	
M1:	Begins completing the table – obtaining correct first row and first column and using symmetry
M1:	Mostly correct – three rows or three columns correct (so demonstrates understanding of using *)
A1:	Completely correct
(a)(ii)	
M1:	States closure and identifies the identity as zero
M1:	Finds inverses for each element
A1:	States that associative law is satisfied and so all axioms satisfied and S is a group
(b)	
M1:	Clearly begins process to find $4*4*4$ reaching $6*4$ or $4*6$ with clear explanation
A1:	Gives answer as zero, states identity and deduces that order is 3
(c)	
M1:	Finds either 3 or 5 or both
A1:	Expresses four of the six terms as powers of either generator correctly (may omit identity and generator itself)
A1:	Expresses all six terms correctly in terms of either 3 or 5 (Do not need to give both)

Question	Scheme	Marks	AOs
5(a)	P_{n-1} is the population at the end of year $n - 1$ and this is increased by 10% by the end of year n , so is multiplied by 110% = 1.1 to give $1.1 \times P_{n-1}$ as new population by natural causes	B1	3.3
	Q is subtracted from $1.1 \times P_{n-1}$ as Q is the number of deer removed from the estate	B1	3.4
	So $P_n = 1.1P_{n-1} - Q$, $P_0 = 5000$ as population at start is 5000 and $n \in \mathbb{Z}^+$	B1	1.1b
		(3)	
(b)	Let $n = 0$, then $P_0 = (5000 - 10Q)(1.1)^0 + 10Q = 5000$ so result is true when $n = 0$	B1	2.1
	Assume result is true for $n = k$, $P_k = (1.1)^k (5000 - 10Q) + 10Q$, then as $P_{k+1} = 1.1P_k - Q$, so $P_{k+1} = \dots$	M1	2.4
	$P_{k+1} = 1.1 \times 1.1^k (5000 - 10Q) + 1.1 \times 10Q - Q$	A1	1.1b
	So $P_{k+1} = (5000 - 10Q)(1.1)^{k+1} + 10Q$,	A1	1.1b
	Implies result holds for $n = k + 1$ and so by induction $P_n = (5000 - 10Q)(1.1)^n + 10Q$, is true for all integer n	B1	2.2a
		(5)	
(c)	For $Q < 500$ the population of deer will grow, for $Q > 500$ the population of deer will fall	B1	3.4
	For $Q = 500$ the population of deer remains steady at 5000,	B1	3.4
		(2)	
(10 marks)			
Notes:			
(a)			
B1: Need to see 10% increase linked to multiplication by scale factor 1.1			
B1: Needs to explain that subtraction of Q indicates the removal of Q deer from population			
B1: Needs complete explanation with mention of $P_n = 1.1P_{n-1} - Q$, $P_0 = 5000$ being the initial number of deer			
(b)			
B1: Begins proof by induction by considering $n = 0$			
M1: Assumes result is true for $n = k$ and uses iterative formula to consider $n = k + 1$			
A1: Correct algebraic statement			
A1: Correct statement for $k + 1$ in required form			
B1: Completes the inductive argument			
(c)			
B1: Consideration of both possible ranges of values for Q as listed in the scheme			
B1: Gives the condition for the steady state			